7-3 Day 4 Volume : Cross Secons

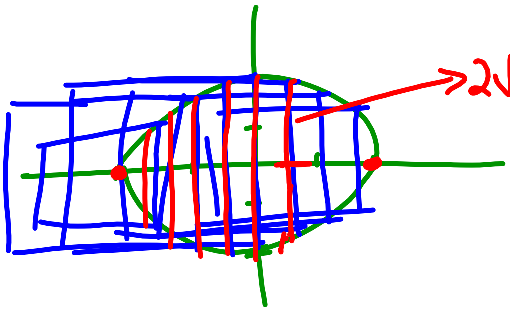
Learning Targets

I can find the volume of a solid that has been created using the cross sections on base method.

Ex1. The base of a solid is the circle $x^2 + y^2 = 4$. Find the volume of

$y = \pm\sqrt{4-x^2}$

a.) The solid with square cross secons.



$A = bh = b^2$

$\int_{-2}^{2} \left(2\sqrt{4-x^2}\right)^2 dx$

$= \int_{-2}^{2} (4(4-x^2))\,dx$

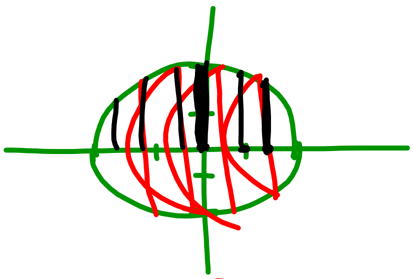
$= 42.667 \text{ units}^3$

$= \int_{-2}^{2} (16-4x^2)\,dx$

$= 16x - \frac{4}{3}x^3 \Big|_{-2}^{2} = 32 - \frac{32}{3} - \left(-32 + \frac{32}{3}\right)$

$64 - \frac{64}{3}$

b.) The solid with semi-circular cross secons.



$x^2 + y^2 = 4$

$r = y = \sqrt{4-x^2}$

$A_{semicircle} = \frac{\pi r^2}{2}$

$\int_{-2}^{2} \frac{\pi \left(\sqrt{4-x^2}\right)^2}{2}\, dx$

$= \int_{-2}^{2} \frac{\pi}{2}(4-x^2)\, dx$

$= 16.755 \text{ units}^3 = \frac{16\pi}{3}$

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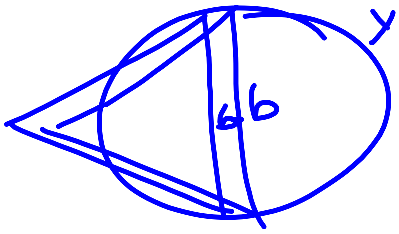
c.) The solid with equilateral triangular cross secons



$x^2+y^2=4$

$y=\sqrt{4-x^2}$

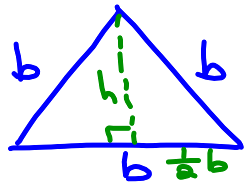
$b=2\sqrt{4-x^2}$

$A=\frac{1}{2}\cdot b\cdot\frac{\sqrt{3}}{2}b$

$=\frac{\sqrt{3}}{4}b^2$

$\int_{-2}^{2}\frac{\sqrt{3}}{4}\left(2\sqrt{4-x^2}\right)^2dx$

$=18.475$ units^3



$A=\frac{1}{2}bh$

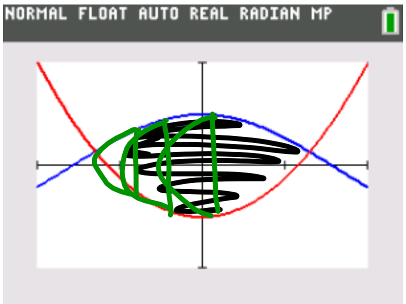
$(\frac{1}{2}b)^2+h^2=b^2$

$\frac{1}{4}b^2+h^2=b^2$

$h^2=\frac{3}{4}b^2$

$h=\frac{\sqrt{3}}{2}b$

Ex2. The base of a solid is the area bounded by the curves y=cos(x) and $y=\frac{3}{4}x^2-1$. Find the volume of the solid with semicircular cross secons.



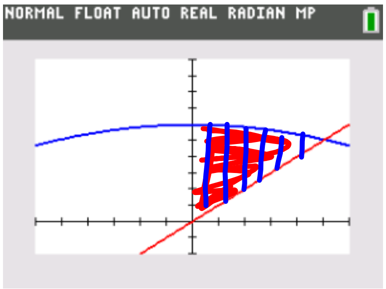
$$A=\frac{\pi r^2}{2}$$

$$r=\frac{\cos x-\left(\frac{3}{4}x^2-1\right)}{2}$$

$$\int_{-1.2999}^{1.2999}\frac{\pi}{2}\left(\frac{\cos x-\left(\frac{3}{4}x^2-1\right)}{2}\right)^2dx$$

$$=2.144\text{ units}^3$$

Ex4 a.) The base of a solid is the area bounded by the curve $f(x) = 6cos\left(\frac{1}{9}x\right)$ and the curve g(x)=x and the y-axis. Find the volume of the solid with square cross secons.



5.07212

$A = b^2$

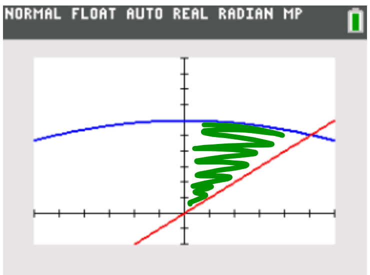
$$\int_0^{5.07212} \left(6\cos\left(\tfrac{1}{9}x\right) - x\right)^2 dx$$

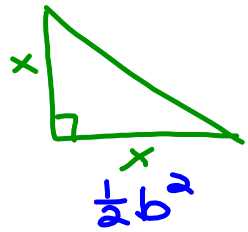
$= 65.636 \text{ units}^3$

b.) What if the cross secons had been isosceles right triangles (with one of the legs on the base) instead?

$y_1 = 6\cos\left(\frac{1}{9}x\right)$

$y_2 = x$





$$\int_0^{5.072} A\,dx$$

$$A = \frac{b^2}{2} \leftarrow b = y_1 - y_2$$

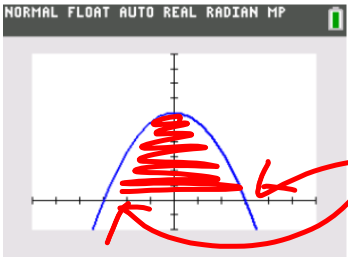
$$\int_0^{5.0721228} \left(\frac{(y_1 - y_2)^2}{2}\right) dx = 32.818$$

Ex4. The base of a solid is the area bounded by the curve $f(x) = 6 - \frac{2}{3}x^2$ and the x-axis. Find the volume of the solid with:

a.) Equilateral Triangular Cross Secons

$A = \frac{\sqrt{3}}{4} b^2$

$y_1 =$ $y_2 = 0$



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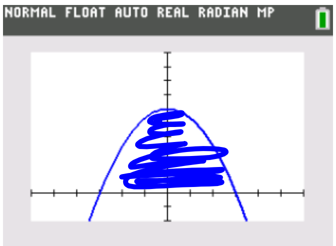
$\int_{-3}^{3} \frac{\sqrt{3}}{4} (y_1)^2 \, dx =$ 49.883 units^3

b.) Isosceles Right Triangular Cross Secons (with the hypotenuse on the base).



$\int_{-3}^{3} \frac{1}{2}\left(6 - \frac{2}{3}x^2\right)^2 dx$

$= 28.8 \, u^3$

$\frac{x}{\sqrt{2}}$

$A = \frac{1}{2} x\sqrt{2} \cdot \frac{x}{\sqrt{2}}$

$= \frac{1}{4} x^2$

Homework

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